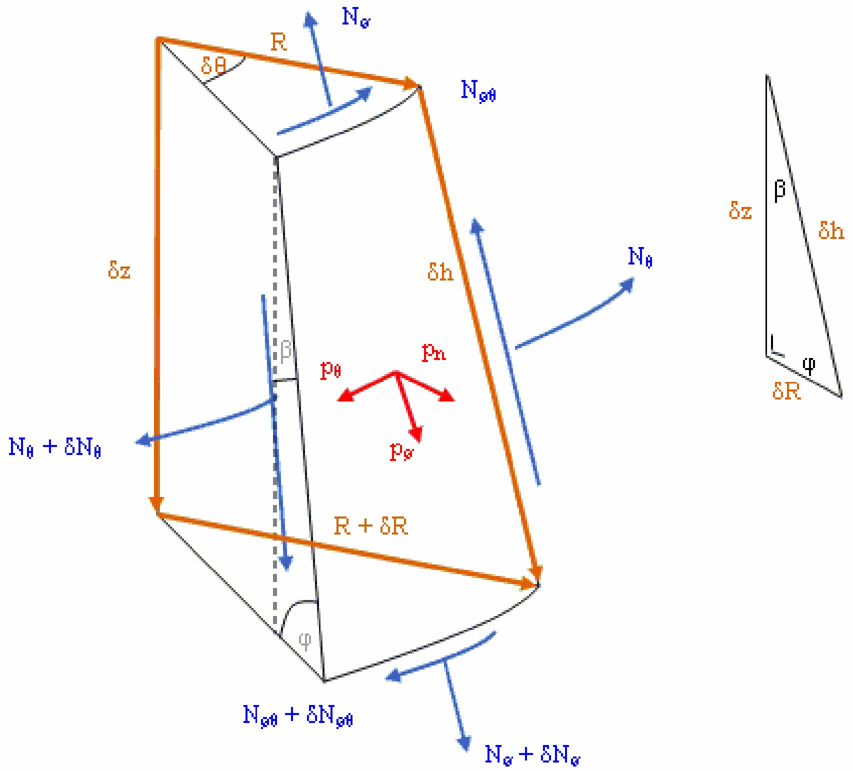

From triangle: $\cos\varphi = \sin\beta$ and $\sin\varphi = \cos\beta$

$$\therefore \partial h = \frac{\partial z}{\sin\varphi} = \frac{\partial z}{\cos\beta} \text{ and } \partial R = \frac{\cos\varphi}{\sin\varphi}\partial z = \frac{\sin\beta}{\cos\beta}\partial z, \text{ hence } R = \frac{\sin\beta}{\cos\beta}z \text{ and } R\partial h = z\partial z\frac{\sin\beta}{\cos\beta}.$$

Use small angle approximation: As $\partial\theta \to 0$, $\cos\partial\theta \to 1$ and $\sin\partial\theta \to \partial\theta$,

and also $\partial\theta^2 \approx 0$, $\partial N_\theta \approx 0$, $\partial N_\phi \approx 0$, $\partial N_{\phi\theta} \approx 0$, $\partial R\partial h \approx 0$.

Normal Equilibrium:

$$p_n(R + \partial R)\partial\theta\partial h = N_\theta\partial\theta\partial h\cos\beta$$

$$p_n R\partial\theta\partial h + p_n\partial R\partial\theta\partial h = N_\theta\partial\theta\partial h\cos\beta$$

$$p_n z\partial z\frac{\sin\beta}{\cos\beta}\partial\theta\frac{1}{\cos\beta} = N_\theta\partial\theta\partial z\frac{\cos\beta}{\cos\beta}$$

$$N_\theta = p_n z\frac{\sin\beta}{\cos^2\beta}$$

$$\therefore N_\theta = p_n z\sec\beta\tan\beta \quad ...(1)$$

Circumferential Equilibrium:

$$N_{\phi\theta}R\partial\theta + N_\theta\partial h = (N_{\phi\theta} + \partial N_{\phi\theta})(R + \partial R)\partial\theta + (N_\theta + \partial N_\theta)\partial h + p_\theta(R + \partial R)\partial\theta\partial h + N_{\phi\theta}\partial h\partial\theta\sin\beta$$

$$\partial N_\theta\partial h + N_{\phi\theta}\partial R\partial\theta + \partial N_{\phi\theta}R\partial\theta + \partial N_{\phi\theta}\partial R\partial\theta + p_\theta R\partial\theta\partial h + N_{\phi\theta}\partial h\partial\theta\sin\beta = 0$$

$$\frac{\partial N_\theta\partial z}{\cos\beta} + \frac{N_{\phi\theta}\partial z\partial\theta\sin\beta}{\cos\beta} + \frac{\partial N_{\phi\theta}z\partial\theta\sin\beta}{\cos\beta} + \frac{\partial N_{\phi\theta}\partial z\partial\theta\sin\beta}{\cos\beta} + \frac{p_\theta z\partial\theta\partial z}{\cos\beta} + \frac{N_{\phi\theta}\partial z\partial\theta\sin\beta}{\cos\beta} = 0$$

$$\frac{\partial N_\theta}{\partial \theta}.\frac{1}{\sin\beta} + N_{\phi\theta} + \frac{z\partial N_{\phi\theta}}{\partial z} + p_\theta z\frac{1}{\cos\beta} + N_{\phi\theta} = 0$$

But from equation 1, $N_\theta = p_n z\frac{\sin\beta}{\cos^2\beta}$ hence $\frac{\partial N_\theta}{\partial \theta} = \frac{z\partial p_n}{\partial \theta}\frac{\sin\beta}{\cos^2\beta}$.

$$2N_{\phi\theta} + \frac{z\partial N_{\phi\theta}}{\partial z} = -z\frac{1}{\cos\beta}(p_\theta + \frac{1}{\cos\beta}z\frac{\partial p_n}{\partial \theta})$$

$$\therefore 2N_{\phi\theta} + \frac{z\partial N_{\phi\theta}}{\partial z} = -z\sec\beta(p_\theta + \sec\beta z\frac{\partial p_n}{\partial \theta}) \quad ...(2)$$

Meridional Equilibrium:

$$N_\phi R\partial\theta + N_{\phi\theta}\partial h + N_\theta \partial h\partial\theta\sin\beta = p_\phi(R+\partial R)\partial\theta\partial h + (N_\phi + \partial N_\phi)(R+\partial R)\partial\theta + (N_{\phi\theta} + \partial N_{\phi\theta})\partial h$$

$$N_\theta\partial h\partial\theta\sin\beta = p_\phi R\partial\theta\partial h + N_\phi\partial R\partial\theta + \partial N_\phi R\partial\theta + \partial N_\phi\partial R\partial\theta + \partial N_{\phi\theta}\partial h$$

$$\frac{N_\theta\partial z\partial\theta\sin\beta}{\cos\beta} = \frac{p_\phi z\partial\theta\partial z\sin\beta}{\cos^2\beta} + \frac{N_\phi\partial z\partial\theta\sin\beta}{\cos\beta} + \frac{\partial N_\phi z\partial\theta\sin\beta}{\cos\beta} + \frac{\partial N_\phi\partial z\partial\theta\sin\beta}{\cos\beta} + \frac{\partial N_{\phi\theta}\partial z}{\cos\beta}$$

$$N_\theta = p_\phi z\frac{1}{\cos\beta} + N_\phi + \frac{z\partial N_\phi}{\partial z} + \partial N_\phi + \frac{\partial N_{\phi\theta}}{\partial\theta}\frac{1}{\sin\beta}$$

$$N_\phi + \frac{z\partial N_\phi}{\partial z} = -\frac{\partial N_{\phi\theta}}{\partial\theta}\frac{1}{\sin\beta} + N_\theta - p_\phi z\frac{1}{\cos\beta} - \partial N_\phi$$

But from equation 1, $N_\theta = p_n z\frac{\sin\beta}{\cos^2\beta}$

$$N_\phi + \frac{z\partial N_\phi}{\partial z} = -\frac{1}{\sin\beta}\frac{\partial N_{\phi\theta}}{\partial\theta} + z\frac{1}{\cos\beta}(p_n\frac{\sin\beta}{\cos\beta} - p_\phi)$$

$$\therefore N_\phi + \frac{z\partial N_\phi}{\partial z} = -\mathrm{cosec}\beta\frac{\partial N_{\phi\theta}}{\partial\theta} + z\sec\beta(p_n\tan\beta - p_\phi) \quad ...(3)$$