



Use small angle approximation: As  $\partial\theta \rightarrow 0$ ,  $\cos\partial\theta \rightarrow 1$  and  $\sin\partial\theta \rightarrow \partial\theta$ , and also  $\partial\theta^2 \approx 0$ ,  $\partial N_\theta \approx 0$ .

Radial Equilibrium:

$$p_n R \partial\theta \cos \frac{\partial\theta}{2} \partial z = (N_\theta + \partial N_\theta) \sin \partial\theta \partial z + p_\theta R \partial\theta \sin \frac{\partial\theta}{2} \partial z + R \partial\theta \sin \frac{\partial\theta}{2} \partial N_{z\theta}$$

$$p_n R \partial\theta \partial z = (N_\theta) \partial\theta \partial z$$

$$\therefore p_n R = N_\theta \quad \dots(1)$$

Circumferential Equilibrium:

$$N_\theta \partial z = R \partial\theta \partial N_{z\theta} \cos \frac{\partial\theta}{2} + p_n R \partial\theta \sin \frac{\partial\theta}{2} \partial z + (N_\theta + \partial N_\theta) \cos \partial\theta \partial z + p_\theta R \partial\theta \cos \frac{\partial\theta}{2} \partial z$$

$$N_\theta \partial z = R \partial\theta \partial N_{z\theta} + (N_\theta + \partial N_\theta) \partial z + p_\theta R \partial\theta \partial z$$

$$0 = R \partial\theta \partial N_{z\theta} + \partial N_\theta \partial z + p_\theta R \partial\theta \partial z$$

$$\frac{\partial N_{z\theta}}{\partial z} + \frac{\partial N_\theta}{R \partial\theta} + p_\theta = 0$$

But from equation 1,  $p_n R = N_\theta$ .

$$\therefore \frac{\partial N_{z\theta}}{\partial z} = -\frac{\partial p_n}{\partial\theta} - p_\theta \quad \dots(2)$$

Axial Equilibrium:

$$N_z R \partial\theta + N_z \partial z = p_z R \partial\theta \partial z + (N_z + \partial N_z) R \partial\theta + (N_{z\theta} + \partial N_{z\theta}) \partial z$$

$$0 = p_z R \partial\theta \partial z + \partial N_z R \partial\theta + \partial N_{z\theta} \partial z$$

$$0 = p_z + \frac{\partial N_z}{\partial z} + \frac{\partial N_{z\theta}}{R \partial\theta}$$

$$\therefore \frac{\partial N_z}{\partial z} = -\frac{\partial N_{z\theta}}{R \partial\theta} - p_z \quad \dots(3)$$